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ADDICTION AND SATIATION *

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This letter develops a general model of rational habit formation. The conditions leading to addiction and satiation are compared, and Stigler and Becker's (1977) discussion of beneficial and harmful addiction is clarified and extended.

1. Introduction

A household commodity is said to be habit forming if it not only provides immediate satisfaction but also affects the marginal utility derived from subsequent consumption. This letter develops a general model of rational habit formation and determines conditions under which it leads to addiction and satiation. (A commodity will be called addictive if its current consumption increases as the habits derived from its previous consumption accumulate. It will be called satiating if the opposite occurs.) Addiction is found to be a consequence of 'adjacent' intertemporal complementarity and satiation of 'distant' intertemporal complementarity. In contrast to Pollak (1970,1976), von Weizsäcker (1976), El-Safty (1976a,b) and Hammond (1976), who model only myopic behavior, I derive results for rational consumers, fully aware of the habit forming effects of their actions. Rational habit formation has been studied by Stigler and Becker (1977), Spinnewyn (1981) and Boyer (1978,1983), but Stigler and Becker's analysis is largely intuitive, and the models of Spinnewyn and Boyer are more restrictive than mine. [1] My analysis extends Stigler and Becker's framework and clarifies their discussion of 'beneficial' and 'harmful' addiction.

2. The model

Consider an individual with instantaneous utility function $U(Y, S)$, where $Y = (Y_1, \ldots, Y_n)$ is a vector of current consumption and $S = (S_1, \ldots, S_m)$ is a vector habits. Net additions to habits are a function of Y and S,

$$\dot{S} = F(Y, S) - DS, \tag{1}$$

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[1] Spinnewyn has utility depend only on 'uncommitted' consumption, the difference between current and habitual levels of consumption. Boyer develops a discrete time model in which instantaneous utility depends only on current consumption and total expenditures (i.e., the standard of living) in the immediately preceding period.

where gross additions, $F=(F^1, \ldots, F^m)$, are non-negative and non-decreasing in (Y, S) and depreciation, D, is a non-negative diagonal matrix with $D_{ii}=\delta_i$. The problem is to choose a non-negative, piecewise continuous consumption path, $Y(\cdot)$, which maximizes lifetime utility,

$$J[Y(\cdot)\,|\,A_0, S_0; 0, T] \equiv \int_0^T e^{-\rho t} U(Y, S)\,dt, \tag{2}$$

subject to (1),

$$\dot{A} = rA - pY, \quad \text{and} \tag{3}$$

$$S(0)=S_0, \qquad A(0)=A_0, \qquad A(T)\geqq 0. \tag{4}$$

A_0 denotes the discounted present value of full wealth. Wealth accumulates at the rate of interest minus current consumption expenditures (3) and is subject to boundary conditions (4).

We assume that both U and F are twice continuously differentiable and that U is concave in (Y, S) and strictly concave and increasing in Y. So that each habit, S_i, may be classified as either 'beneficial' or 'harmful' depending on whether $\partial U/\partial S_i$ is positive or negative, we assume

A.1. With respect to each component of S, utility is either strictly increasing or strictly decreasing,

A.2. Gross additions to one component of S are increasing in another only if both increase or both decrease utility. [2]

And we also assume

A.3. F^i is concave if S_i is beneficial and convex if S_i is harmful.

Leonard (1981) has investigated the costate signs and sufficiency conditions for a general class of problems satisfying A.1–A.3.

Introducing costate (shadow price) variables λ and $\mu=(\mu_1, \ldots, \mu_m)$, we write the current-valued Hamiltonian,

$$H \equiv U(Y, S) + [F(Y, S) - DS]\mu + [rA - pY]\lambda. \tag{5}$$

Along an optimal path the Hamiltonian must be maximized at each date with respect to the control variables. For simplicity, let us rule out corner solutions by assuming that $\lim_{y_i \to 0} \partial U/\partial Y_i = \infty$ uniformly for all i. Hence, letting Y, S and A subscripts denote vector partial derivatives,

$$H_Y = U_Y + F_Y\mu - p\lambda = 0, \tag{6}$$

the other necessary conditions are the costate equations,

$$\dot{\lambda} = \rho\lambda - H_A = (\rho - r)\lambda, \qquad \dot{\mu} = \rho\mu - H_S = (\rho I + D - F_S)\mu - U_S \tag{7),(8}$$

('I' is the identity matrix), and the transversality conditions,

$$\lambda(T)A(T) = 0, \qquad \mu(T)S(T) = 0 \tag{9}$$

(which imply $A(T)=0$ and $\mu(T)=0$). Eq. (6) is the familiar requirement that marginal costs and benefits must be equal along an optimal path. Hence, for commodities that generate no habits

[2] These assumptions are not particularly restrictive. Although each component of S is either strictly beneficial or strictly harmful, individual commodities may jointly augment both types of components.

($F_{Y_i} \equiv 0$) we obtain the standard expression: $U_{Y_i} = p_i\lambda$. Habit formation adds to this expression a new term, $F_{Y_i}\mu$, the future utility generated by current consumption, and so drives a wedge between immediate marginal utility and direct marginal cost. We note the following basic results: [3]

(i) If a particular habit, S_i, is beneficial (harmful) the corresponding shadow price, μ_i, will be positive (negative). Moreover, λ is non-negative and, unless all commodities generate harmful habits, strictly positive.
(ii) When $\rho = \delta_i = 0$, $\dot{\mu}_i < 0$ if S_i is beneficial and $\dot{\mu}_i > 0$ if S_i is harmful.
(iii) Any path which satisfies the necessary conditions (6)–(9) and the constraints (1), (3) and (4) is optimal.
(iv) Any optimal consumption path will be continuous and differentiable with respect to time.

3. Discussion

We wish to determine whether and when habit formation induces addiction in rational consumers. Inducing addiction in myopic consumers is, of course, straightforward: just let a commodity generate habits which raise the marginal utility of its consumption relative to other commodities. But it is less clear that a rational consumer can get hooked, particularly on commodities that generate only harmful habits. Stigler and Becker (1977) have claimed that a model similar to the one above can generate rational addiction. In particular, they asserted, but did not prove, that commodities generating harmful habits (which they call 'consumption capital') would be addictive only if their demand is 'sufficiently inelastic' and that those generating beneficial habits would be addictive only if their demand was 'sufficiently elastic' (1977, p. 81). We examine these claims below.

Since H_{YY} is negative definite and Y time-differentiable [result (iv)], we may differentiate eq. (6) with respect to time to obtain

$$\dot{Y} = -H_{YY}^{-1}H_{YS}\dot{S} - H_{YY}^{-1}F_Y\dot{\mu} + H_{YY}^{-1}(p\dot{\lambda} + \dot{p}\lambda). \tag{10}$$

This matrix expression cannot generally be signed, but it simplifies considerably when utility is additive and habits are 'commodity-specific'. [Habits are said to be commodity-specific if for each S_i there exists a unique commodity, call it Y_i, such that F^i and U_{Y_i}/U_{S_i} depend only on (Y_i, S_i).] In this special but important case all the matrices above are diagonal. Hence, combining (10) and (i) yields the following proposition which proves that rational consumers can become addicted to commodities with harmful side effects, and, like their myopic counterparts, are most readily hooked on those generating habits that strongly complement further consumption. We assume that $\dot{p} = 0$ and $r = \rho$ $(= 0)$ so that consumption is constant in the absence of habit formation effects. Then Y_i is addictive (respectively, satiating) if $\dot{Y}_i$ and $\dot{S}_i$ have the same (opposite) sign, i.e., if the consumption of Y_i increases (decreases) as S_i increases, ceterus paribus.

Proposition 1. If habits are commodity-specific, utility additive in the commodity-habit pairs (Y_i, S_i), *and* $r = \rho = \delta_i = \dot{p} = F^i_{Y_iS_i} = 0$, *then*

$$\frac{\partial^2 U}{\partial Y_i \partial S_i} \begin{cases} \leqq 0 \text{ is sufficient for satiation} \\ > 0 \text{ is necessary for addiction} \end{cases} \quad \text{when } S_i \text{ is beneficial, and}$$

$$\frac{\partial^2 U}{\partial Y_i \partial S_i} \begin{cases} < 0 \text{ is necessary for satiation} \\ \geqq 0 \text{ is sufficient for addiction} \end{cases} \quad \text{when } S_i \text{ is harmful.}$$

[3] Results (i) and (iii) follow from Leonard (1981). Proofs of (ii) and (iv), and an alternative proof of (i) are in an appendix available from the author.

The result is immediately applicable to Pollak (1970,1976), Spinnewyn (1981), and others who assume subutility functions of the form $V^i[a(Y_i) + b(S_i)]$ (reflecting the notion that habits alter 'needs'). Specifically:

Corollary 1. If, in addition to the assumptions of Proposition 1, Y_i and S_i enter utility in additive form $a(Y_i) + b(S_i)$, then Y_i will be addictive if and only if S_i is harmful and satiating if and only if S_i is beneficial.

On the other hand, a multiplicative specification is more appropriate when, as Stigler and Becker (1977) suggest, consumption alters the productivity of time and goods devoted to specific activities. Letting utility be a function of household commodities $Z_i = a(Y_i)b(S_i)$, we have:

Corollary 2. Assume that in addition to the assumptions of Proposition 1, Y_i and S_i enter utility in the multiplicative form $Z = a(Y_i)b(S_i)$. Let $\sigma \equiv -U_Z/ZU_{ZZ}$. Then, when habits are beneficial, $\sigma > 1$ is necessary for addiction and $\sigma \leq 1$ is sufficient for satiation. And, when habits are harmful, $\sigma > 1$ is necessary for satiation and $\sigma \leq 1$ is sufficient for addiction.

Corollary 2 clarifies Stigler and Becker's (1977) claim that beneficial addiction arises only when commodity demand is 'sufficiently elastic', but harmful addiction only when it is 'sufficiently inelastic'. It demonstrates that the relevant elasticity is not the comparative-static own-price elasticity of demand, but rather an *intertemporal* elasticity of commodity substitution as measured by σ. [4]

Extending Proposition 1 to non-zero rates of interest, time preference and depreciation, we find addiction to be the consequence of 'adjacent' complementarity and satiation the consequence of 'distant' complementarity:

Proposition 2. If habits are commodity-specific, utility additive in the commodity-habit pairs (Y_i, S_i), and $\dot{p} = 0$, $r = \rho$, and $\delta_i > 0$, then, in the neighborhood of a stable steady-state,

$$\operatorname{sgn}(\dot{Y}_i) = \operatorname{sgn}(-\gamma_i)\operatorname{sgn}(\dot{S}_i), \quad \textit{where}$$

$$\gamma_i \equiv 2\left(\frac{\rho}{2} + \left(\delta_i - F^i_{S_i}\right)\right)\left(\frac{H_{Y_iS_i}}{H_{Y_iY_i}}\right) + F^i_{Y_i}\left(\frac{H_{S_iS_i}}{H_{Y_iY_i}}\right),$$

and $-\gamma_i$ is positive (negative) when the consumption of Y_i is characterized by adjacent (distant) complementarity.

The proof is in an appendix available from the author and in Iannaccone (1984). Note that adjacent complementarity exists if increasing the consumption of the commodity at some date t raises the marginal utility from that commodity at nearby dates relative to distant dates. Under distant complementarity, the situation is reversed [Ryder and Heal (1972, p. 5)].

References

Boyer, Marcel, 1978, A habit forming optimal growth model, International Economic Review 19, 585–609.
Boyer, Marcel, 1983, Rational demand and expenditures patterns under habit formation, Journal of Economic Theory 31, 27–53.

[4] To see that σ measures intertemporal substitutability of Z, rewrite the model in discrete time [max $\Sigma(1+\rho)^{-t}U(Z_t,\ldots)$, etc.]. Then σ is the direct elasticity of substitution between Z_t and $Z_{t'}$ evaluated at the point $Z_t = Z_{t'}$. The continuous case follows by extension.

El-Safty, Ahmad E., 1976a, Adaptive behavior, demand and preferences, Journal of Economic Theory 13, 298–318.
El-Safty, Ahmad E., 1976b, Adaptive behavior and the existence of Weizsäcker's long-run indifference curves, Journal of Economic Theory 13, 319–328.
Hammond, Peter J., 1976, Endogenous tastes and stable long run choice, Journal of Economic Theory 13, 329–340.
Iannaccone, Laurence R., 1984, Consumption capital and habit formation, with an application to religious participation, PhD dissertation (University of Chicago, Chicago, IL).
Leonard, Daniel, 1981, The signs of the co-state variables and sufficiency conditions in a class of optimal control problems, Economics Letters 10, 321–325.
Pollak, Robert A., 1970, Habit formation and dynamic demand functions, Journal of Political Economy 78, July–Aug., 748–763.
Pollak, Robert A., 1976, Habit formation and long-run utility functions, Journal of Economic Theory 13, 272–297.
Ryder, Harl E., Jr. and Geoffrey M. Heal, 1973, Optimal growth with intertemporally dependent preferences, Review of Economic Studies 40, Jan., 1–33.
Spinnewyn, Frans, 1981, Rational habit formation, European Economic Review 15, 91–109.
Stigler, George J. and Gary S. Becker, 1977, De gustibus non est disputandum, American Economic Review 67, March, 76–90.
Von Weizsäcker, Carl Christian, 1976, Notes on endogenous change of tastes, Journal of Economic Theory 13, 345–372.